

## Y11 to Y12 Mathematics Summer Independent Learning

### June to August

Please read the following instructions very carefully and ensure you label and collate all your work ready for checking in September.

For your first maths lesson please bring

- A large A4 folder with five subject dividers.
- These instructions with the tables filled in (print out/copy the tables onto A4 paper).
- Dated and titled work done on each of the topics listed in Task 1 & 3.
- The two practice initial tests (Task 2), fully marked and reviewed.
- A list of questions you need to ask prior to doing your initial test.

### Task 1: Preparation for A level Maths

1. You should spend approximately 1 hour on each topic
2. For each topic, [work through video](#).
3. Complete worksheet **using the technique and layout** used in the video.
4. Make sure you title and date your work.
5. Mark and correct work.
6. Do improvement work as necessary.
7. Repeat for each topic.
8. Keep track by filling in the following table.
9. Collate your work for each topic together so it is easy to check in September. *(See point 3!)*

| Topic                     | <a href="#">Video(s)</a><br><i>(Tick)</i> | Worksheet<br><i>(Tick)</i> | Details of Improvement Work Completed |
|---------------------------|---|----------------------------|---------------------------------------|
| B1 Indices                |   |                            |                                       |
| B2 Surds                  |   |                            |                                       |
| B3 Quadratics             |   |                            |                                       |
| B4 Simultaneous Equations |   |                            |                                       |
| B5 Inequalities           |   |                            |                                       |
| E1 Triangle Geometry      |   |                            |                                       |

## Task 2

1. Do Practice Initial Test 1 under exam conditions.
2. Mark and correct your test and identify any improvement work necessary.
3. Fill in the review sheet below.
4. Revisit relevant videos and worksheets.
5. Update review sheet with details of work completed.

| Topic                     | Score | Improvement Work to Do | Tick |
|---------------------------|-------|------------------------|------|
| B1 Indices                | 11    |                        |      |
| B2 Surds                  | 10    |                        |      |
| B3 Quadratics             | 49    |                        |      |
| B4 Simultaneous Equations | 11    |                        |      |
| B5 Inequalities           | 11    |                        |      |
| E1 Triangle Geometry      | 12    |                        |      |
| Total                     | 114   |                        |      |

6. Do Practice Initial Test 2 under exam conditions.
7. Mark and correct your test and identify any improvement work necessary.
8. Fill in the review sheet below.
9. Revisit relevant videos and worksheets.
10. Update review sheet with details of work completed.
11. Make a list of questions you need to ask prior to doing your initial test for real!

| Topic                     | Score | Improvement Work to Do | Tick | Questions to ask... |
|---------------------------|-------|------------------------|------|---------------------|
| B1 Indices                | 11    |                        |      |                     |
| B2 Surds                  | 10    |                        |      |                     |
| B3 Quadratics             | 49    |                        |      |                     |
| B4 Simultaneous Equations | 11    |                        |      |                     |
| B5 Inequalities           | 11    |                        |      |                     |
| E1 Triangle Geometry      | 12    |                        |      |                     |
| Total                     | 114   |                        |      |                     |

## Video hyperlinks

### B1 Indices

<https://youtu.be/1lThXgU08S0>

<https://youtu.be/v5bn4HZrmQs>

<https://youtu.be/W0h4rHj88ys>

### B2 Surds

<https://youtu.be/jHelde32YtI>

### B3 Quadratics

<https://youtu.be/Pziws8ojnlk>

[https://youtu.be/sn\\_joGVj15w](https://youtu.be/sn_joGVj15w)

<https://youtu.be/kk7p6hjn7hQ>

[https://youtu.be/tolqbX\\_NXHo](https://youtu.be/tolqbX_NXHo)

### B4 Simultaneous Equations

<https://youtu.be/4SRtwS5unwE>

### B5 Inequalities

[https://youtu.be/wDut-In\\_7Wg](https://youtu.be/wDut-In_7Wg)

### E1 Triangle Geometry

<https://youtu.be/uVI6TAb0vBg>

Exam Questions (OCR/MEI C1 Questions)

|    |   |
|----|---|
| 1. | <p><b>Jan 05 Q5</b><br/>Find the value of the following.</p> <p>(i) <math>\left(\frac{1}{3}\right)^{-2}</math> [2]</p> <p>(ii) <math>16^{\frac{3}{4}}</math> [2]</p>  |
| 2. | <p><b>June 05 Q6</b><br/>Simplify the following.</p> <p>(i) <math>a^0</math> [1]</p> <p>(ii) <math>a^6 \div a^{-2}</math> [1]</p> <p>(iii) <math>(9a^6b^2)^{-\frac{1}{2}}</math> [3]</p>  |
| 3. | <p><b>June 06 Q9</b><br/>Simplify the following.</p> <p>(i) <math>\frac{16^{\frac{1}{2}}}{81^{\frac{1}{3}}}</math> [2]</p> <p>(ii) <math>\frac{12(a^3b^2c)^4}{4a^2c^6}</math> [3]</p>   |
| 4. | <p><b>Jan 07 Q6</b><br/>Find the value of each of the following, giving each answer as an integer or fraction as appropriate.</p> <p>(i) <math>25^{\frac{3}{2}}</math> [2]</p> <p>(ii) <math>\left(\frac{7}{3}\right)^{-2}</math> [2]</p> |
| 5. | <p><b>June 07 Q5</b></p> <p>(i) Find <math>a</math>, given that <math>a^3 = 64x^{12}y^3</math>. [2]</p> <p>(ii) Find the value of <math>\left(\frac{1}{2}\right)^{-5}</math>. [2]</p>   |

## Indices Exam Questions Solutions

1. Jan 05 Q5

$$(i) \left(\frac{1}{3}\right)^{-2}$$

$$= \left(\frac{3}{1}\right)^2$$

$$= 9$$

$$(ii) 16^{3/4}$$

$$= (16^{1/4})^3$$

$$= 2^3$$

$$= 8$$

2. Jan 05 Q6

$$(i) a^0 = 1 \quad (ii) a^6 \div a^{-2} = a^8$$

$$(iii) (9a^6b^2)^{-1/2} = \frac{1}{3} a^{-3} b^{-1}$$

$$\text{or } \frac{1}{3a^3b}$$

3. June 06 Q9

$$(i) \frac{16^{1/2}}{81^{3/4}}$$

$$= \frac{4}{(81^{1/4})^3}$$

$$= \frac{4}{3^3}$$

$$= \frac{4}{27}$$

$$(ii) \frac{12(a^3b^2c)^4}{4a^2c^6}$$

$$= \frac{12a^{12}b^8c^4}{4a^2c^6}$$

$$= 3a^{10}b^8c^{-2}$$

$$\text{or } \frac{3a^{10}b^8}{c^2}$$

4. Jan 07 Q6

$$(i) 25^{3/2} = (25^{1/2})^3$$

$$= 5^3$$

$$= 125$$

$$(ii) \left(\frac{7}{3}\right)^{-2} = \left(\frac{3}{7}\right)^2$$

$$= \frac{9}{49}$$

5. June 07 Q5

$$(i) a^3 = 64x^{12}y^3$$

$$a = (64x^{12}y^3)^{1/3}$$

$$a = 4x^4y$$

$$(ii) \left(\frac{1}{2}\right)^{-5}$$

$$= \left(\frac{2}{1}\right)^5$$

$$= 32$$

## Exam Questions (AQA Questions)

|    |  |
|----|--|
| 1. | <p><b>Jan 05 Q5</b></p> <p>(a) Simplify <math>(\sqrt{12} + 2)(\sqrt{12} - 2)</math>. <span style="float: right;"><i>(2 marks)</i></span></p> <p>(b) Express <math>\sqrt{12}</math> in the form <math>m\sqrt{3}</math>, where <math>m</math> is an integer. <span style="float: right;"><i>(1 mark)</i></span></p> <p>(c) Express <math>\frac{\sqrt{12} + 2}{\sqrt{12} - 2}</math> in the form <math>a + b\sqrt{3}</math>, where <math>a</math> and <math>b</math> are integers. <span style="float: right;"><i>(4 marks)</i></span></p>  |
| 2. | <p><b>June 05 Q5</b></p> <p>Express each of the following in the form <math>m + n\sqrt{3}</math>, where <math>m</math> and <math>n</math> are integers:</p> <p>(a) <math>(\sqrt{3} + 1)^2</math>; <span style="float: right;"><i>(2 marks)</i></span></p> <p>(b) <math>\frac{\sqrt{3} + 1}{\sqrt{3} - 1}</math>. <span style="float: right;"><i>(3 marks)</i></span></p>   |
| 3. | <p><b>Jan 06 Q1</b></p> <p>(a) Simplify <math>(\sqrt{5} + 2)(\sqrt{5} - 2)</math>. <span style="float: right;"><i>(2 marks)</i></span></p> <p>(b) Express <math>\sqrt{8} + \sqrt{18}</math> in the form <math>n\sqrt{2}</math>, where <math>n</math> is an integer. <span style="float: right;"><i>(2 marks)</i></span></p>  |
| 4. | <p><b>June 06 Q4</b></p> <p>(a) Express <math>(4\sqrt{5} - 1)(\sqrt{5} + 3)</math> in the form <math>p + q\sqrt{5}</math>, where <math>p</math> and <math>q</math> are integers. <span style="float: right;"><i>(3 marks)</i></span></p> <p>(b) Show that <math>\frac{\sqrt{75} - \sqrt{27}}{\sqrt{3}}</math> is an integer and find its value. <span style="float: right;"><i>(3 marks)</i></span></p>  |
| 5. | <p><b>Jan 07 Q3</b></p> <p>(a) Express <math>\frac{\sqrt{5} + 3}{\sqrt{5} - 2}</math> in the form <math>p\sqrt{5} + q</math>, where <math>p</math> and <math>q</math> are integers. <span style="float: right;"><i>(4 marks)</i></span></p> <p>(b) (i) Express <math>\sqrt{45}</math> in the form <math>n\sqrt{5}</math>, where <math>n</math> is an integer. <span style="float: right;"><i>(1 mark)</i></span></p> <p>(ii) Solve the equation</p> $x\sqrt{20} = 7\sqrt{5} - \sqrt{45}$ <p>giving your answer in its simplest form. <span style="float: right;"><i>(3 marks)</i></span></p> |
| 6. | <p><b>June 07 Q7</b></p> <p>(a) Express <math>\frac{\sqrt{63}}{3} + \frac{14}{\sqrt{7}}</math> in the form <math>n\sqrt{7}</math>, where <math>n</math> is an integer. <span style="float: right;"><i>(3 marks)</i></span></p> <p>(b) Express <math>\frac{\sqrt{7} + 1}{\sqrt{7} - 2}</math> in the form <math>p\sqrt{7} + q</math>, where <math>p</math> and <math>q</math> are integers. <span style="float: right;"><i>(4 marks)</i></span></p>   |

## Exam Questions Solutions - Surds

1. Jan 05 Q5

$$\begin{aligned}
 \text{(a)} \quad & (\sqrt{12} + 2)(\sqrt{12} - 2) \quad (M1) \\
 & = 12 - 2\sqrt{12} + 2\sqrt{12} - 4 \\
 & = 8 \quad (A1) \\
 \text{(b)} \quad & \sqrt{12} = \sqrt{4 \times 3} \\
 & = 2\sqrt{3} \quad (B1) \\
 \text{(c)} \quad & \frac{(\sqrt{12} + 2)(\sqrt{12} + 2)}{(\sqrt{12} - 2)(\sqrt{12} + 2)} \quad (M1) \\
 & = \frac{12 + 2\sqrt{12} + 2\sqrt{12} + 4}{8} \quad (A1) \\
 & = \frac{16 + 4\sqrt{12}}{8} \\
 & = \frac{16 + 8\sqrt{3}}{8} \quad (A1) \\
 & = 2 + \sqrt{3} \quad (A1)
 \end{aligned}$$

2. June 05 Q5

$$\begin{aligned}
 \text{(a)} \quad & (\sqrt{3} + 1)^2 \\
 & = (\sqrt{3} + 1)(\sqrt{3} + 1) \quad (M1) \\
 & = 3 + \sqrt{3} + \sqrt{3} + 1 \\
 & = 4 + 2\sqrt{3} \quad (A1) \\
 \text{(b)} \quad & \frac{(\sqrt{3} + 1)(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)} \quad (M1) \\
 & = \frac{4 + 2\sqrt{3}}{3 + \sqrt{3} - \sqrt{3} - 1} \quad (A1) \\
 & = \frac{4 + 2\sqrt{3}}{2} \\
 & = 2 + \sqrt{3} \quad (A1)
 \end{aligned}$$

3. Jan 06 Q1

$$\begin{aligned}
 \text{(a)} \quad & (\sqrt{5} + 2)(\sqrt{5} - 2) \\
 & = 5 - 2\sqrt{5} + 2\sqrt{5} - 4 \quad (M1) \\
 & = 1 \quad (A1) \\
 \text{(b)} \quad & \sqrt{8} + \sqrt{18} \\
 & = \sqrt{4}\sqrt{2} + \sqrt{9}\sqrt{2} \quad (M1) \\
 & = 2\sqrt{2} + 3\sqrt{2} \quad (A1) \\
 & = 5\sqrt{2}
 \end{aligned}$$

4. June 06 Q4

$$\begin{aligned}
 \text{(a)} \quad & (4\sqrt{5} - 1)(\sqrt{5} + 3) \\
 & = 20 + 12\sqrt{5} - \sqrt{5} - 3 \quad (M1)(A1) \\
 & = 17 + 11\sqrt{5} \quad (A1) \\
 \text{(b)} \quad & \frac{\sqrt{75} - \sqrt{27}}{\sqrt{3}} \\
 & = \frac{5\sqrt{3} - 3\sqrt{3}}{\sqrt{3}} \quad (M1) \\
 & = \frac{2\sqrt{3}}{\sqrt{3}} \quad (M1) \\
 & = 2 \quad (A1)
 \end{aligned}$$



5. Jan 07 Q3

$$(a) \frac{(\sqrt{5}+3)(\sqrt{5}+2)}{(\sqrt{5}-2)(\sqrt{5}+2)} \quad (M1) \quad (b)(i) \sqrt{45} = \sqrt{9 \cdot 5} \quad (B1)$$
$$= 3\sqrt{5}$$

$$= \frac{5+2\sqrt{5}+3\sqrt{5}+6}{5-4} \quad (A1) \quad (ii) x\sqrt{20} = 7\sqrt{5} - \sqrt{45} \quad (M1)$$

$$= 11 + 5\sqrt{5} \quad (A1) \quad 2x\sqrt{5} = 7\sqrt{5} - 3\sqrt{5} \quad (M1)$$
$$2x\sqrt{5} = 4\sqrt{5} \quad (M1)$$
$$2x = 4 \quad (A1)$$
$$x = 2 \quad (A1)$$

6. June 07 Q7

$$(a) \frac{\sqrt{63}}{3} + \frac{14}{\sqrt{7}} \quad (M1) \quad (b) \frac{(\sqrt{7}+1)(\sqrt{7}+2)}{(\sqrt{7}-2)(\sqrt{7}+2)} \quad (M1)$$

$$= \frac{3\sqrt{7}}{3} + \frac{14 \cdot \sqrt{7}}{\sqrt{7} \cdot \sqrt{7}} \quad (M1) \quad = \frac{7+2\sqrt{7}+\sqrt{7}+2}{7-4} \quad (A1)$$

$$= \frac{3\sqrt{7}}{3} + \frac{14\sqrt{7}}{7} \quad = \frac{9+3\sqrt{7}}{3} \quad (A1)$$

$$= \sqrt{7} + 2\sqrt{7} \quad = 3 + \sqrt{7} \quad (A1)$$

$$= 3\sqrt{7} \quad (A1)$$



Exam Questions (AQA C1 Questions)

|           |  |
|-----------|--|
| <p>1.</p> | <p><b>Jan 2011 Q7</b></p> <p>(a) (i) Express <math>4 - 10x - x^2</math> in the form <math>p - (x + q)^2</math>. <span style="float: right;">(2 marks)</span></p> <p>(ii) Hence write down the equation of the line of symmetry of the curve with equation <math>y = 4 - 10x - x^2</math>. <span style="float: right;">(1 mark)</span></p>  |
| <p>2.</p> | <p><b>June 11 Q4</b></p> <p>(a) Express <math>x^2 + 5x + 7</math> in the form <math>(x + p)^2 + q</math>, where <math>p</math> and <math>q</math> are rational numbers. <span style="float: right;">(3 marks)</span></p> <p>(b) A curve has equation <math>y = x^2 + 5x + 7</math>.</p> <p>(i) Find the coordinates of the vertex of the curve. <span style="float: right;">(2 marks)</span></p> <p>(ii) State the equation of the line of symmetry of the curve. <span style="float: right;">(1 mark)</span></p> <p>(iii) Sketch the curve, stating the value of the intercept on the <math>y</math>-axis. <span style="float: right;">(3 marks)</span></p> <p>(c) Describe the geometrical transformation that maps the graph of <math>y = x^2</math> onto the graph of <math>y = x^2 + 5x + 7</math>. <span style="float: right;">(3 marks)</span></p>  |
| <p>3.</p> | <p><b>Jan 12 Q2</b></p> <p>(a) Factorise <math>x^2 - 4x - 12</math>. <span style="float: right;">(1 mark)</span></p> <p>(b) Sketch the graph with equation <math>y = x^2 - 4x - 12</math>, stating the values where the curve crosses the coordinate axes. <span style="float: right;">(4 marks)</span></p> <p>(c) (i) Express <math>x^2 - 4x - 12</math> in the form <math>(x - p)^2 - q</math>, where <math>p</math> and <math>q</math> are positive integers. <span style="float: right;">(2 marks)</span></p> <p>(ii) Hence find the minimum value of <math>x^2 - 4x - 12</math>. <span style="float: right;">(1 mark)</span></p> <p>(d) The curve with equation <math>y = x^2 - 4x - 12</math> is translated by the vector <math>\begin{bmatrix} -3 \\ 2 \end{bmatrix}</math>.<br/>Find an equation of the new curve. You need not simplify your answer. <span style="float: right;">(2 marks)</span></p> |
| <p>4.</p> | <p><b>June 12 Q5</b></p> <p>(a) (i) Express <math>x^2 - 3x + 5</math> in the form <math>(x - p)^2 + q</math>. <span style="float: right;">(2 marks)</span></p> <p>(ii) Hence write down the equation of the line of symmetry of the curve with equation <math>y = x^2 - 3x + 5</math>. <span style="float: right;">(1 mark)</span></p>   |

|    |  |
|----|--|
| 5. | <p><b>Jan 13 Q4</b></p> <p>(a) (i) Express <math>x^2 - 6x + 11</math> in the form <math>(x - p)^2 + q</math>. <span style="float: right;"><i>(2 marks)</i></span></p> <p>(ii) Use the result from part (a)(i) to show that the equation <math>x^2 - 6x + 11 = 0</math> has no real solutions. <span style="float: right;"><i>(2 marks)</i></span></p> <p>(b) A curve has equation <math>y = x^2 - 6x + 11</math>.</p> <p>(i) Find the coordinates of the vertex of the curve. <span style="float: right;"><i>(2 marks)</i></span></p> <p>(ii) Sketch the curve, indicating the value of <math>y</math> where the curve crosses the <math>y</math>-axis. <span style="float: right;"><i>(3 marks)</i></span></p> <p>(iii) Describe the geometrical transformation that maps the curve with equation <math>y = x^2 - 6x + 11</math> onto the curve with equation <math>y = x^2</math>. <span style="float: right;"><i>(3 marks)</i></span></p> |
| 6. | <p><b>June 13 Q5</b></p> <p>(a) (i) Express <math>2x^2 + 6x + 5</math> in the form <math>2(x + p)^2 + q</math>, where <math>p</math> and <math>q</math> are rational numbers. <span style="float: right;"><i>(2 marks)</i></span></p> <p>(ii) Hence write down the minimum value of <math>2x^2 + 6x + 5</math>. <span style="float: right;"><i>(1 mark)</i></span></p>   |

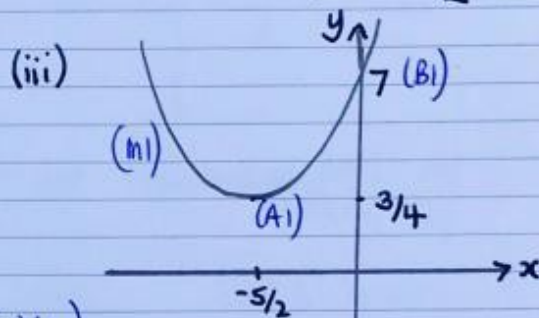
## Quadratics Exam Questions Solutions

### 1. Jan 2011 Q7

(a)(i)  $4 - 10x - x^2$  (ii) line of symmetry  $x = -5$  (B1 FT)  
 $\equiv -(x^2 + 10x - 4)$   
 $\equiv -((x+5)^2 - 25 - 4)$  (M1)  
 $\equiv -((x+5)^2 - 29)$   
 $\equiv 29 - (x+5)^2$  (A1)

### 2. June 11 Q4

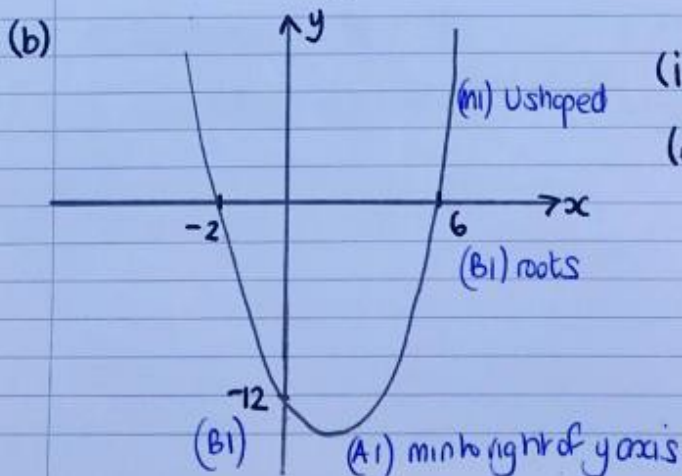
(a)  $x^2 + 5x + 7 \equiv$  (b)(i) when  $x = -\frac{5}{2}$   $y = \frac{3}{4}$  (M1)  
 $\equiv (x + \frac{5}{2})^2 - \frac{25}{4} + \frac{28}{4}$  (M1)(M1)  $\therefore$  vertex at  $(-\frac{5}{2}, \frac{3}{4})$  (A1)  
 $\equiv (x + \frac{5}{2})^2 + \frac{3}{4}$  (A1) (ii) line of symmetry  $x = -\frac{5}{2}$  (B1 FT)



(c) Translation  $(-\frac{5}{2}, \frac{3}{4})$  (M1)(A1)  
(E1)

### 3. Jan 12 Q2

(a)  $x^2 - 4x - 12$   
 $\equiv (x-6)(x+2)$  (B1)



(c)(i)  $x^2 - 4x - 12$   
 $\equiv (x-2)^2 - 4 - 12$  (M1)  
 $\equiv (x-2)^2 - 16$  (A1)

(ii) min value is -16 (B1 FT)

(d)  $y = (x+3)^2 - 4(x+3) - 12 + 2$  (M1)  
 $y = (x+3)^2 - 4(x+3) - 10$   
or any equivalent form (A1)

i.e.  $y = (x+1)^2 - 14$



#### 4. June 12 Q5

(a)(i)  $x^2 - 3x + 5$

$$\equiv (x - 3/2)^2 - \frac{9}{4} + \frac{20}{4}$$

$$\equiv (x - 3/2)^2 + \frac{11}{4} \quad (A_1)$$

(ii) line of symmetry is  $x = 3/2$  (B1 FT)

#### 5. Jan 13 Q4

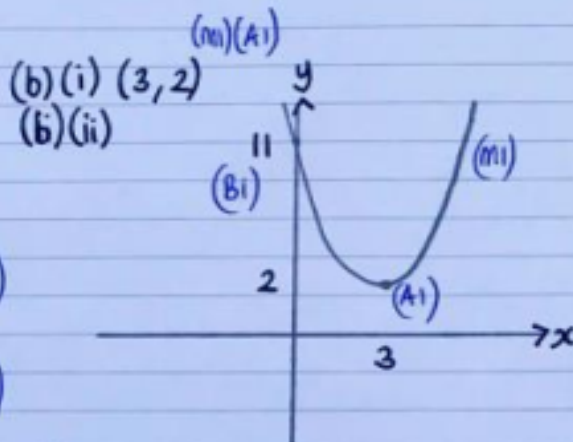
(a)(i)  $x^2 - 6x + 11$

$$\equiv (x - 3)^2 - 9 + 11$$

$$\equiv (x - 3)^2 + 2 \quad (A_1)$$

(ii)  $(x - 3)^2 + 2 = 0$   
 $(x - 3)^2 = -2$  (M1)

can't take square root of negative number and get real solutions.  $\therefore$  no real solutions (A1)



(iii) Translation  $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$  (M1)  
(E1) (A1)

Gone backwards!

#### 6. June 13 Q5

(a)(i)  $2x^2 + 6x + 5$

$$\equiv 2[x^2 + 3x] + 5$$

$$\equiv 2[(x + 3/2)^2 - 9/4] + 5$$

$$\equiv 2(x + 3/2)^2 - \frac{9}{2} + \frac{10}{2}$$

$$\equiv 2(x + 3/2)^2 + \frac{1}{2} \quad (A_1)$$

or  $2x^2 + 6x + 5$

$$\equiv 2[x^2 + 3x + 9/2]$$

$$\equiv 2[(x + 3/2)^2 - 9/4 + 10/4]$$

$$\equiv 2[(x + 3/2)^2 + 1/4]$$

$$\equiv 2(x + 3/2)^2 + 1/2$$

(ii) Min value is  $y = 1/2$  (B1 FT)

**Topic B4** Simultaneous equations**Exam Questions** (AQA C1 Questions)

|           |   |
|-----------|---|
| <b>1</b>  | Solve the simultaneous equations<br>$y - 3x + 2 = 0$ $y^2 - x - 6x^2 = 0$ <p style="text-align: right;"><b>(Total 7 marks)</b></p>  |
| <b>2</b>  | The curve $C$ has equation $y = x^2 - 4$ and the straight line $l$ has equation $y + 3x = 0$ .<br><p>(a) In the space below, sketch <math>C</math> and <math>l</math> on the same axes. <span style="float: right;"><b>(3)</b></span></p> <p>(b) Write down the coordinates of the points at which <math>C</math> meets the coordinate axes. <span style="float: right;"><b>(2)</b></span></p> <p>(c) Using algebra, find the coordinates of the points at which <math>l</math> intersects <math>C</math>. <span style="float: right;"><b>(4)</b></span></p> <p style="text-align: right;"><b>(Total 9 marks)</b></p> |
| <b>3</b>  | <b>Jan 011 Q7</b><br><p>(b) The curve <math>C</math> has equation <math>y = 4 - 10x - x^2</math> and the line <math>L</math> has equation <math>y = k(4x - 13)</math>, where <math>k</math> is a constant.</p> <p>(i) Show that the <math>x</math>-coordinates of any points of intersection of the curve <math>C</math> with the line <math>L</math> satisfy the equation</p> $x^2 + 2(2k + 5)x - (13k + 4) = 0$ <p style="text-align: right;"><i>(1 mark)</i></p>   |
| <b>4.</b> | <b>Jan 13 Q8</b><br>A curve has equation $y = 2x^2 - x - 1$ and a line has equation $y = k(2x - 3)$ , where $k$ is a constant.<br><p>(a) Show that the <math>x</math>-coordinate of any point of intersection of the curve and the line satisfies the equation</p> $2x^2 - (2k + 1)x + 3k - 1 = 0$ <p style="text-align: right;"><i>(1 mark)</i></p>  |

## Simultaneous Equations Exam Questions

1)  $y = 3x - 2$

$$(3x - 2)^2 - x - 6x^2 = 0$$

$$9x^2 - 12x + 4 - x - 6x^2 = 0$$

$$3x^2 - 13x + 4 = 0$$

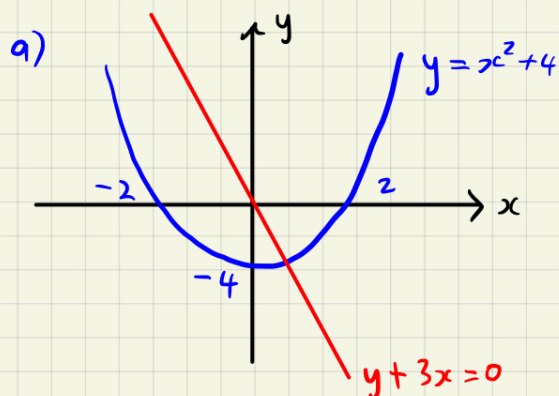
$$(3x - 1)(x - 4) = 0$$

$$x = \frac{1}{3}$$
$$y = -1$$

or

$$x = 4$$
$$y = 10$$

2)



b)  $(2, 0)$ ,  $(-2, 0)$

c)  $y = -3x$

$$-3x = x^2 - 4$$

$$x^2 + 3x - 4 = 0$$

$$(x - 1)(x + 4) = 0$$

$$x = 1 \quad \text{or} \quad x = -4$$
$$y = -3 \quad \quad \quad y = 12$$

intersections are  $(1, -3)$ ,  $(-4, 12)$

Jan 11 Q7

$$y = 4 - 10x - x^2 \quad y = R(4x - 13)$$

$$R(4x - 13) = 4 - 10x - x^2$$

$$x^2 + 10x + 4Rx - 13R - 4 = 0$$

$$x^2 + 2(2R+5)x - (13R+4) = 0$$

Jan 13 Q8

$$y = 2x^2 - x - 1 \quad y = R(2x - 3)$$

$$2x^2 - x - 1 = R(2x - 3)$$

$$2x^2 - x - 1 = 2Rx - 3R$$

$$2x^2 + 2Rx - x + 3R - 1 = 0$$

$$2x^2 - (2R+1)x + 3R - 1 = 0$$



Topic: B5 Inequalities

Exam Questions (AQA C1 Questions)

|    |  |
|----|--|
| 1. | <p><b>Jan 11 Q7</b></p> <p>(iii) Solve the inequality <math>4k^2 + 33k + 29 &gt; 0</math>. <span style="float: right;">(4 marks)</span></p>  |
| 2. | <p><b>June 11 Q7</b></p> <p>Solve each of the following inequalities:</p> <p>(a) <math>2(4 - 3x) &gt; 5 - 4(x + 2)</math>; <span style="float: right;">(2 marks)</span></p> <p>(b) <math>2x^2 + 5x \geq 12</math>. <span style="float: right;">(4 marks)</span></p>  |
| 3. | <p><b>Jan 12 Q6</b></p> <p>A rectangular garden is to have width <math>x</math> metres and length <math>(x + 4)</math> metres.</p> <p>(a) The perimeter of the garden needs to be greater than 30 metres.</p> <p>Show that <math>2x &gt; 11</math>. <span style="float: right;">(1 mark)</span></p> <p>(b) The area of the garden needs to be less than 96 square metres.</p> <p>Show that <math>x^2 + 4x - 96 &lt; 0</math>. <span style="float: right;">(1 mark)</span></p> <p>(c) Solve the inequality <math>x^2 + 4x - 96 &lt; 0</math>. <span style="float: right;">(4 marks)</span></p> <p>(d) Hence determine the possible values of the width of the garden. <span style="float: right;">(1 mark)</span></p> |
| 4. | <p><b>June 12 Q7a</b></p> <p>(ii) Solve the inequality <math>3x^2 - 10x + 8 &lt; 0</math>. <span style="float: right;">(4 marks)</span></p>  |

## Inequalities Exam Questions Solutions

1. Jan 11 Q7 (ii)

$$4R^2 + 33R + 29 > 0$$

$$(4R + 29)(R + 1) > 0 \quad (m_1)$$

$$\text{cvs at } R = -\frac{29}{4} \quad R = -1 \quad (A_1)$$



$$R < -\frac{29}{4} \text{ or } R > -1 \quad (A_1)$$

4

2. June 11 Q7

$$(a) \quad 2(4 - 3x) > 5 - 4(x + 2)$$

$$8 - 6x > 5 - 4x - 8 \quad (m_1)$$

$$-2x > -11$$

$$2x < 11$$

$$x < \frac{11}{2} \quad (A_1)$$

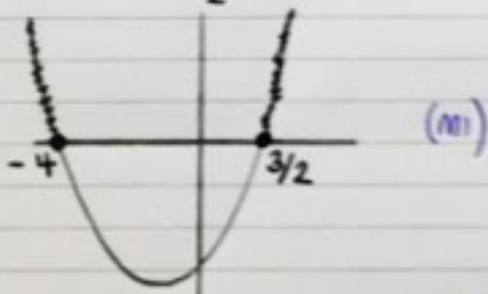
(x-1 reverse inequality sign)

$$(b) \quad 2x^2 + 5x \geq 12$$

$$2x^2 + 5x - 12 \geq 0$$

$$(2x - 3)(x + 4) \geq 0 \quad (m_1)$$

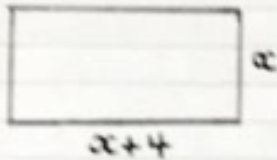
$$\text{cvs at } x = \frac{3}{2} \quad x = -4 \quad (A_1)$$



$$x \leq -4 \text{ or } x \geq \frac{3}{2} \quad (A_1)$$

6

3 Jan 12 Q6



$$\begin{aligned} \text{(a)} \quad x+x+x+4+x+4 &> 30 \\ 4x+8 &> 30 \\ 4x &> 22 \\ 2x &> 11 \quad (\text{B}_1) \\ (x > \frac{11}{2}) \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad x(x+4) &< 96 \\ x^2+4x &< 96 \\ x^2+4x-96 &< 0 \quad (\text{B}_1) \\ (x+12)(x-8) &< 0 \quad (\text{M}_1) \\ \text{cvs } x &= -12 \quad x=8 \quad (\text{A}_1) \end{aligned}$$



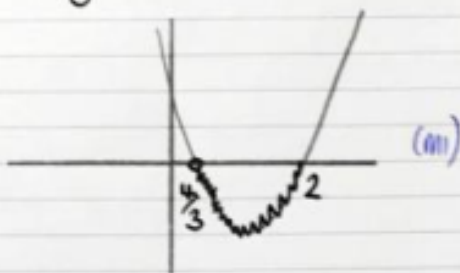
$$-12 < x < 8 \quad (\text{A}_1)$$

$$\text{(c)} \quad \frac{11}{2} < x < 8 \quad (\text{B}_1)$$

4. June 12 Q7(a)

$$\begin{aligned} 3x^2-10x+8 &< 0 \\ (3x-4)(x-2) &< 0 \quad (\text{M}_1) \end{aligned}$$

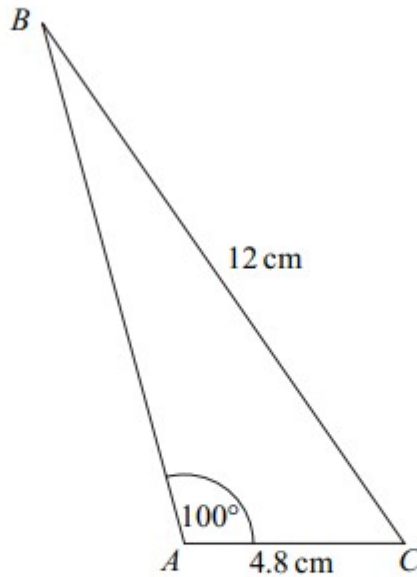
$$\text{cvs } x = \frac{4}{3} \quad x=2 \quad (\text{A}_1)$$



$$\frac{4}{3} < x < 2 \quad (\text{A}_1)$$

1. June 2006 Q2

The diagram shows a triangle  $ABC$ .



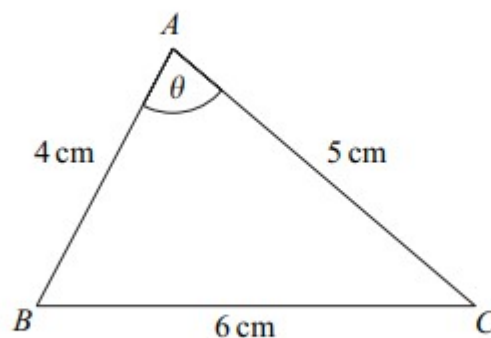
The lengths of  $AC$  and  $BC$  are 4.8 cm and 12 cm respectively.

The size of the angle  $BAC$  is  $100^\circ$ .

- (a) Show that angle  $ABC = 23.2^\circ$ , correct to the nearest  $0.1^\circ$ . (3 marks)
- (b) Calculate the area of triangle  $ABC$ , giving your answer in  $\text{cm}^2$  to three significant figures. (3 marks)

2. Jan 2007 Q4 (adapted)

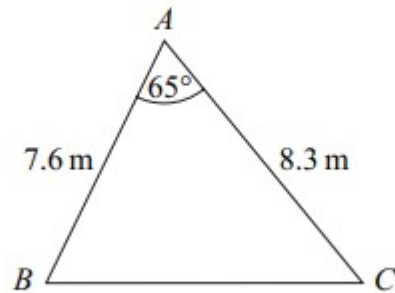
The triangle  $ABC$ , shown in the diagram, is such that  $BC = 6$  cm,  $AC = 5$  cm and  $AB = 4$  cm. The angle  $BAC$  is  $\theta$ .



- (a) Use the cosine rule to show that  $\cos \theta = \frac{1}{8}$ . (3 marks)
- (c) Hence find the area of the triangle  $ABC$ . (2 marks)

3. June 2008 Q4

The diagram shows a triangle  $ABC$ .



The size of angle  $BAC$  is  $65^\circ$ , and the lengths of  $AB$  and  $AC$  are 7.6 m and 8.3 m respectively.

(a) Show that the length of  $BC$  is 8.56 m, correct to three significant figures. (3 marks)

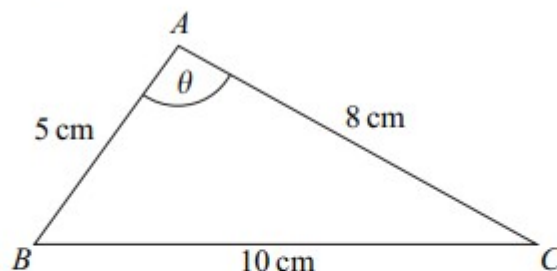
(b) Calculate the area of triangle  $ABC$ , giving your answer in  $\text{m}^2$  to three significant figures. (2 marks)

(c) The perpendicular from  $A$  to  $BC$  meets  $BC$  at the point  $D$ .

Calculate the length of  $AD$ , giving your answer to the nearest 0.1 m. (3 marks)

4. Jan 2011 Q3

The triangle  $ABC$ , shown in the diagram, is such that  $AB = 5$  cm,  $AC = 8$  cm,  $BC = 10$  cm and angle  $BAC = \theta$ .



(a) Show that  $\theta = 97.9^\circ$ , correct to the nearest  $0.1^\circ$ . (3 marks)

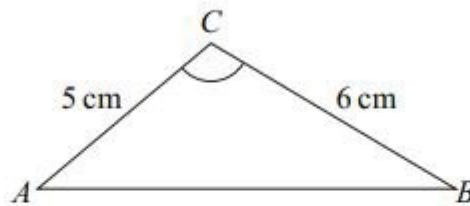
(b) (i) Calculate the area of triangle  $ABC$ , giving your answer, in  $\text{cm}^2$ , to three significant figures. (2 marks)

(ii) The line through  $A$ , perpendicular to  $BC$ , meets  $BC$  at the point  $D$ . Calculate the length of  $AD$ , giving your answer, in cm, to three significant figures. (3 marks)



5. Jan 2013 Q3

The diagram shows a triangle  $ABC$ .



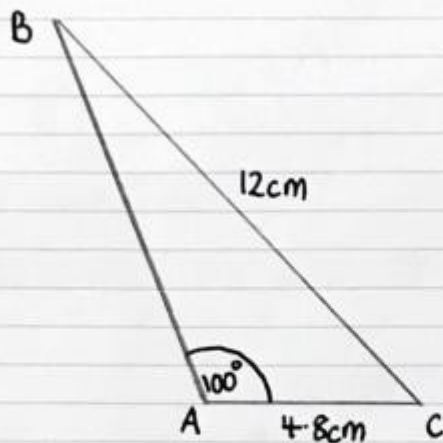
The lengths of  $AC$  and  $BC$  are 5 cm and 6 cm respectively.

The area of triangle  $ABC$  is  $12.5 \text{ cm}^2$ , and angle  $ACB$  is **obtuse**.

- (a) Find the size of angle  $ACB$ , giving your answer to the nearest  $0.1^\circ$ . (3 marks)
- (b) Find the length of  $AB$ , giving your answer to two significant figures. (3 marks)

### Triangle Geometry Exam Questions

1. June 2006 Q2



$$(a) \frac{\sin \hat{A}BC}{4.8} = \frac{\sin 100}{12} \quad (M_1)$$

$$\sin \hat{A}BC = 0.4 \sin 100 \quad (M_1)$$

$$\hat{A}BC = \sin^{-1}(0.4 \sin 100)$$

$$\hat{A}BC = 23.19882755$$

$$\hat{A}BC = 23.2 \text{ correct to } (A_1) \\ \text{nearest } 0.1^\circ$$

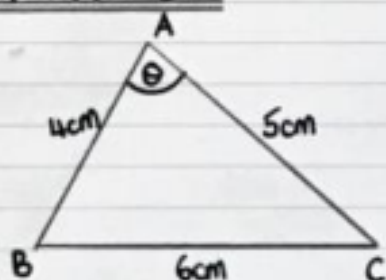
$$(b) \text{ angle } \hat{A}CB = 180 - 100 - 23.2 \\ = 56.8^\circ \quad (M_1)$$

$$\text{area of } \Delta = \frac{1}{2} \times 4.8 \times 12 \times \sin 56.8 \quad (M_1)$$

$$= 24.1 \text{ cm}^2 \text{ to 3.s.f.} \quad (A_1)$$

6

2. Jan 2007 Q4



(a) Using cosine rule (M1)

$$6^2 = 4^2 + 5^2 - 2(4)(5)\cos\theta$$

$$36 = 41 - 40\cos\theta$$

$$40\cos\theta = 5 \quad (M1)$$

$$\cos\theta = \frac{5}{40}$$

$$\cos\theta = \frac{1}{8} \quad (A1)$$

(c)  $\theta = \cos^{-1}\left(\frac{1}{8}\right)$

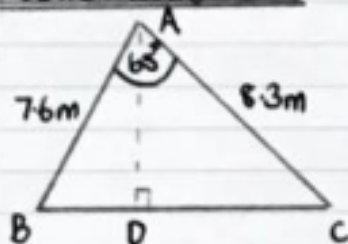
$$\theta = 82.8^\circ \quad (M1)$$

$$A = \frac{1}{2} \times 4 \times 5 \times \sin 82.8^\circ$$

$$\text{Area} = 9.92 \text{ cm}^2 \text{ to 3.s.f.} \quad (A1)$$

5

3. June 2008 Q4



(a) Using cosine rule

$$BC^2 = 7.6^2 + 8.3^2 - 2(7.6)(8.3)\cos 65^\circ \quad (M1)$$

$$BC^2 = 73.33248\dots \quad (M1)$$

$$BC = 8.5634\dots$$

$$BC = 8.56 \text{ m to 3.s.f.} \quad (A1)$$

(b)  $A = \frac{1}{2} \times 7.6 \times 8.3 \sin 65^\circ \quad (M1)$

$$= 28.6 \text{ m}^2 \text{ to 3.s.f.} \quad (A1)$$

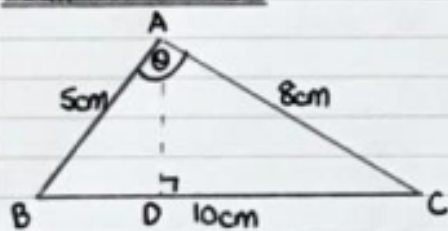
(c)  $28.6 = \frac{1}{2} \times 8.56 \times AD \quad (M1)(M1)$

$$AD = 6.7 \text{ m to nearest 0.1 m} \quad (A1)$$

8



4. Jan 2011 Q3



(a) Using cosine rule

$$10^2 = 5^2 + 8^2 - 2(5)(8) \cos \theta \quad (M1)$$

$$100 = 89 - 80 \cos \theta \quad (M1)$$

$$80 \cos \theta = -11$$

$$\cos \theta = \frac{-11}{80}$$

$$\theta = 97.9032\dots$$

$$\theta = 97.9^\circ \text{ to nearest } 0.1^\circ \quad (A1)$$

$$(b) (i) \text{ Area} = \frac{1}{2} \times 5 \times 8 \times \sin 97.9^\circ \quad (M1)$$

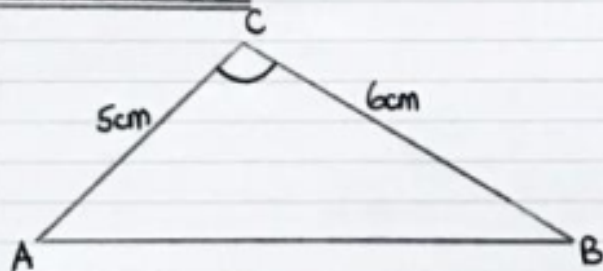
$$= 19.8 \text{ cm}^2 \text{ to 3.s.f.} \quad (A1)$$

$$(ii) 19.8 = \frac{1}{2} \times 10 \times AD \quad (M1)(M1)$$

$$AD = 3.96 \text{ cm to 3.s.f.} \quad (A1)$$

8

5. Jan 2013 Q3



$$(a) 12.5 = \frac{1}{2} \times 5 \times 6 \times \sin C \quad (M1)$$

$$\frac{12.5}{15} = \sin C \quad (A1)$$

$$C = 56.4^\circ$$

but  $\hat{A}CB$  is obtuse

$$\therefore \hat{A}CB = 180 - 56.4$$

$$= 123.6^\circ \quad (A1)$$

(b) Using cosine rule

$$AB^2 = 5^2 + 6^2 - 2(5)(6) \cos 123.6^\circ \quad (M1)(M1)$$

$$AB^2 = 94.203\dots$$

$$AB = 9.7 \text{ cm to 2.s.f.} \quad (A1)$$

6

## TASK 2

### Year 12 Initial Test for Mathematics

Write out the solutions to each of the following questions.  
Show full working, **without** the use of a calculator.

#### Practice 1 (No Calculator)

##### B1 Indices

|   |   |                         |                           |
|---|---|-------------------------|---------------------------|
| 1. Evaluate $\left(\frac{8}{125}\right)^{-2/3}$ | 2. Express in the form $x^k$<br>$\frac{\sqrt{x} \times \sqrt[3]{x}}{x^2}$ | 3. Solve $9^{x-2} = 27$ | 4. Solve $16^x = 4^{1-x}$ |
|---|---|-------------------------|---------------------------|

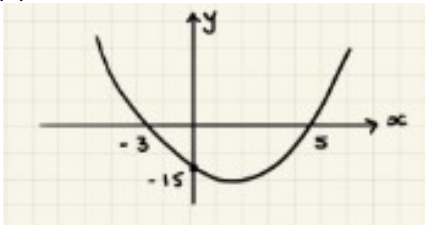
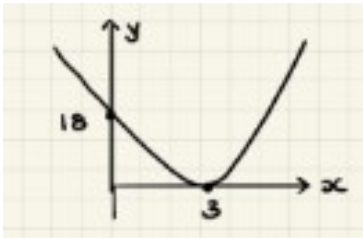
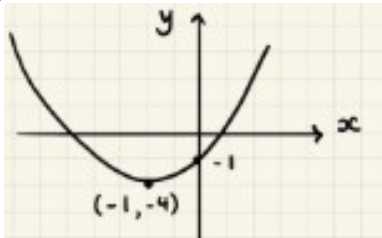
##### B2 Surds

|                         |   |  |  |
|-------------------------|---|--|--|
| 1. Simplify $\sqrt{72}$ | 2. Expand and simplify $(2\sqrt{7} - 5\sqrt{3})(3\sqrt{7} + 4\sqrt{3})$ | 3. Rationalise the denominator<br>$\frac{11}{2\sqrt{5}}$ | 4. Rationalise the denominator<br>$\frac{8 - 3\sqrt{5}}{2 + \sqrt{5}}$ |
|-------------------------|---|--|--|

##### B3 Quadratics

|   |                                    |                                       |
|---|------------------------------------|---------------------------------------|
| 1. Solve the following quadratic equations by factorising and use your solutions to sketch the related quadratic graph, labelling all intersections with the coordinate axis. |                                    |                                       |
| (a) (i) $x^2 + 3x - 28 = 0$   | (b) (i) $x^2 - 6x + 9 = 0$         | (c) (i) $2x^2 - 21x + 27 = 0$         |
| (a) (ii) Sketch $y = x^2 + 3x - 28$   | (b) (ii) Sketch $y = x^2 - 6x + 9$ | (c) (ii) Sketch $y = 2x^2 - 21x + 27$ |

|   |   |  |
|---|---|--|
| 2. Solve the following quadratic equations by completing the square and use your solutions to sketch the related quadratic graph, labelling all intersections with the coordinate axis and turning point. |   |  |
| (a) (i) $x^2 + 4x - 7 = 0$  | (b) (i) $11 + 8x - x^2 = 0$                                     | (c) (i) $3x^2 - 12x + 2 = 0$                                     |
| (ii) Write $y = x^2 + 4x - 7$ in the form $y = a(x + b)^2 + c$  | (ii) Write $y = 11 + 8x - x^2$ in the form $y = a(x + b)^2 + c$ | (ii) Write $y = 3x^2 - 12x + 2$ in the form $y = a(x + b)^2 + c$ |
| (iii) Sketch $y = x^2 + 4x - 7$   | (iii) Sketch $y = 11 + 8x - x^2$                                | (iii) Sketch $y = 3x^2 - 12x + 2$                                |

|  |  |  |
|--|--|--|
| 3. Evaluate the equation of the following quadratics, giving your answer in the form $y = ax^2 + bx + c$ |  |  |
| (a)<br>               | (b)<br> | (c)<br> |

**B4 Simultaneous Equations**

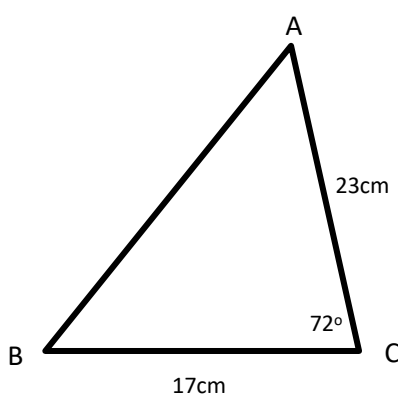
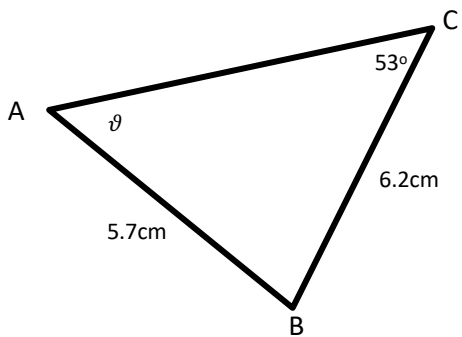
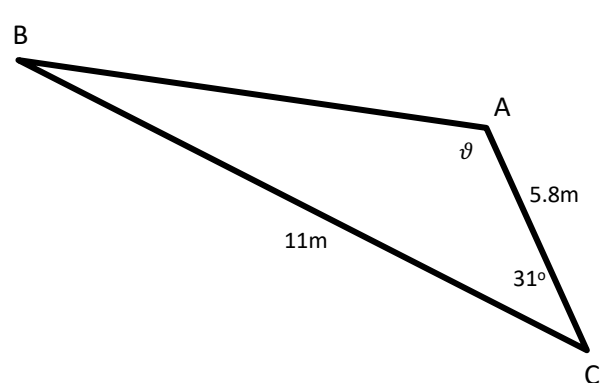
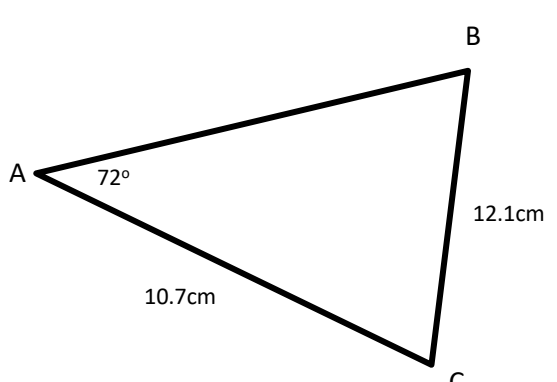
|  |  |  |
|--|--|--|
| <b>1.</b> Solve<br>$\begin{aligned} 3x + 3y &= -4 \\ 5x - 2y &= 5 \end{aligned}$ | <b>2.</b> Solve<br>$\begin{aligned} y &= x - 6 \\ \frac{1}{2}x - y &= 4 \end{aligned}$ | <b>3.</b> Solve<br>$\begin{aligned} 3x^2 - x - y^2 &= 0 \\ x + y &= 1 \end{aligned}$ |
|--|--|--|

**B5 Inequalities**

Find the set of values for which...

|                                  |                                  |                                |
|----------------------------------|----------------------------------|--------------------------------|
| <b>1.</b> $3(1 - 2t) \leq t - 4$ | <b>2.</b> $2x^2 - 9x + 4 \leq 0$ | <b>3.</b> $2y + 3 < 3y(y - 2)$ |
|----------------------------------|----------------------------------|--------------------------------|

**E1 Triangle Geometry (Calculator)**

|   |  |
|---|--|
| <b>1.</b> Calculate the length AB<br>                                   | <b>2.</b> Calculate the angle $\vartheta$<br>         |
| <b>3.</b> Calculate the length AB and the obtuse angle $\vartheta$<br> | <b>4.</b> Calculate the area of the triangle ABC<br> |

## Practice 1

### B1 Indices

$$\begin{aligned} 1. & \left(\frac{8}{125}\right)^{-2/3} \\ &= \left(\frac{125}{8}\right)^{2/3} \\ &= \left(\frac{5}{2}\right)^2 \quad \text{M1} \\ &= \frac{25}{4} \quad \text{A1} \end{aligned}$$

$$\begin{aligned} 2. & \frac{\sqrt{x} \times 3\sqrt{x}}{x^2} \\ &= \frac{x^{1/2} \times x^{1/3}}{x^2} \quad \text{M1} \\ &= \frac{x^{5/6}}{x^2} \quad \text{A1} \\ &= x^{-7/6} \quad \text{A1} \end{aligned}$$

$$\begin{aligned} 3. & 9^{x-2} = 27 \\ & (3^2)^{x-2} = 3^3 \quad \text{M1} \\ & 3^{2x-4} = 3^3 \\ & 2x-4 = 3 \quad \text{M1} \\ & 2x = 7 \\ & x = 7/2 \quad \text{A1} \end{aligned}$$

$$\begin{aligned} 4. & 16^x = 4^{1-x} \\ & (4^2)^x = 4^{1-x} \quad \text{M1} \\ & 4^{2x} = 4^{1-x} \quad \text{M1} \\ & 2x = 1-x \quad \text{M1} \\ & 3x = 1 \Rightarrow x = 1/3 \quad \text{A1} \end{aligned}$$

### B2 Surds

$$\begin{aligned} 1. & \sqrt{72} \\ &= \sqrt{36 \times 2} \\ &= 6\sqrt{2} \quad \text{A1} \end{aligned}$$

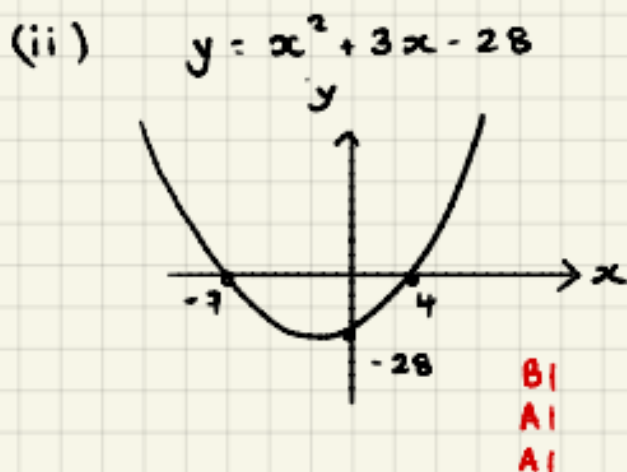
$$\begin{aligned} 2. & (2\sqrt{7} - 5\sqrt{3})(3\sqrt{7} + 4\sqrt{3}) \\ & 42 + 8\sqrt{21} - 15\sqrt{21} - 60 \quad \text{M1 A1} \\ & -7\sqrt{21} - 18 \quad \text{A1} \end{aligned}$$

$$\begin{aligned} 3. & \frac{11}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \quad \text{M1} \\ &= \frac{11\sqrt{5}}{10} \quad \text{A1} \end{aligned}$$

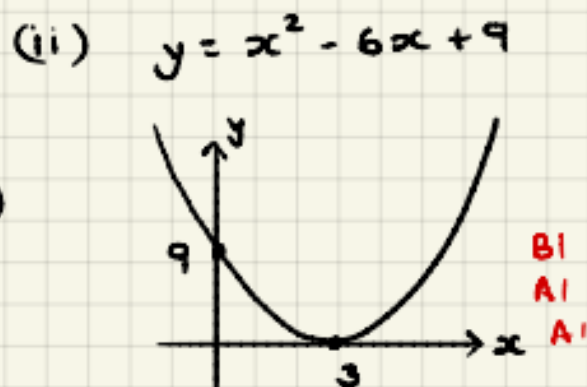
$$\begin{aligned} 4. & \frac{8-3\sqrt{5}}{2+\sqrt{5}} \times \frac{(2-\sqrt{5})}{(2-\sqrt{5})} \quad \text{M1} \\ &= \frac{16-8\sqrt{5}-6\sqrt{5}+15}{4-5} \quad \text{A1} \\ &= \frac{31-14\sqrt{5}}{-1} = 14\sqrt{5} - 31 \quad \text{A1} \end{aligned}$$

### B3 Quadratics

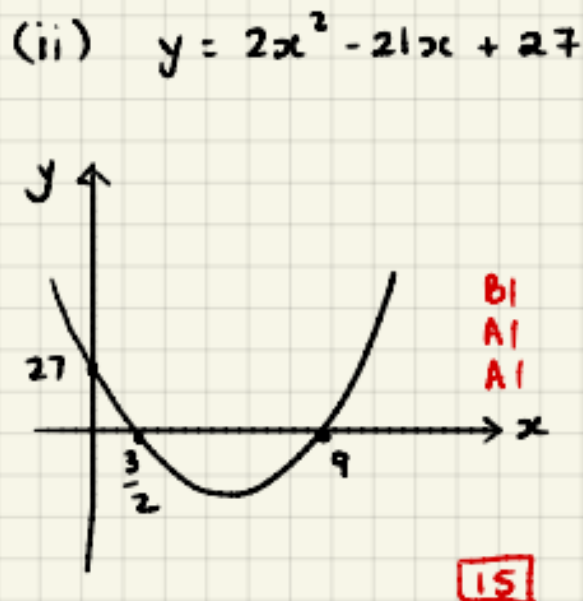
1. (a) (i)  $x^2 + 3x - 28 = 0$   
 $(x+7)(x-4) = 0$  M1  
 $x = -7$  or  $x = 4$  A1



(b) (i)  $x^2 - 6x + 9 = 0$   
 $(x-3)^2 = 0$  M1  
A1  $x = 3$  (repeated)



(c) (i)  $2x^2 - 21x + 27 = 0$   
 $(2x-3)(x-9) = 0$  M1  
 $x = 3/2$   $x = 9$  A1



B1 shape, location related to axes

A1 intersections x-axis

A1 intersections y-axis

$$2. (a) (i) x^2 + 4x - 7 = 0$$

$$(x+2)^2 - 4 - 7 = 0 \text{ M1}$$

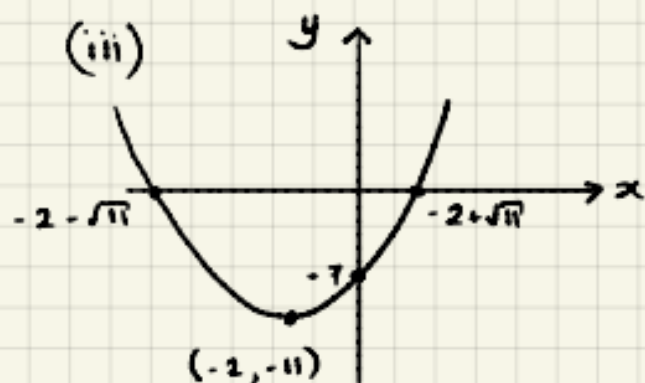
$$(x+2)^2 = 11$$

$$x+2 = \pm\sqrt{11}$$

$$x = -2 \pm \sqrt{11} \text{ A1}$$

$$(ii) y = x^2 + 4x - 7$$

$$y = (x+2)^2 - 11 \text{ B1}$$



Graphs

B1 Shape

A1 Vertex

A1 Intersections x-axis

A1 Intersections y-axis

$$(b) (i) 11 + 8x - x^2 = 0$$

$$-(x^2 - 8x - 11) = 0 \text{ M1}$$

$$-[(x-4)^2 - 16 - 11] = 0 \text{ M1}$$

$$-(x-4)^2 + 27 = 0$$

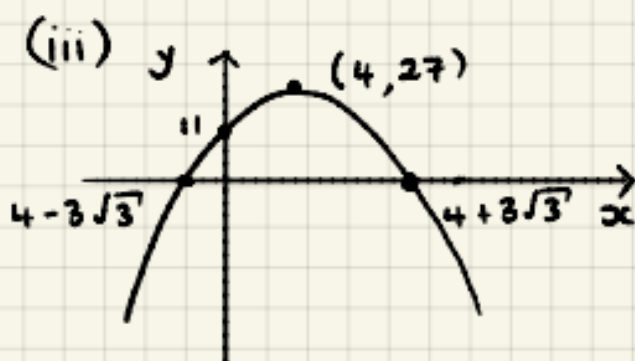
$$(x-4)^2 = 27$$

$$x-4 = \pm 3\sqrt{3}$$

$$x = 4 \pm 3\sqrt{3} \text{ A1}$$

$$(ii) y = 11 + 8x - x^2$$

$$y = 27 - (x-4)^2 \text{ B1}$$



$$(c) (i) 3x^2 - 12x + 2 = 0$$

$$3\left[x^2 - 4x + \frac{2}{3}\right] = 0 \text{ M1}$$

$$3\left[(x-2)^2 - 4 + \frac{2}{3}\right] = 0 \text{ M1}$$

$$3\left[(x-2)^2 - \frac{10}{3}\right] = 0$$

$$3(x-2)^2 - 10 = 0$$

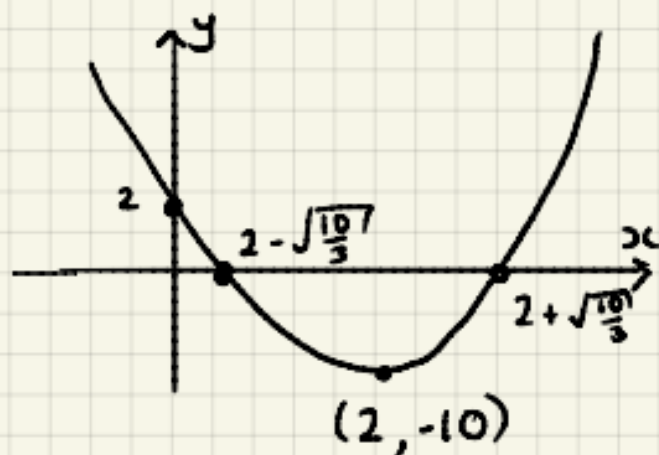
$$(x-2)^2 = \frac{10}{3}$$

$$x-2 = \pm\sqrt{\frac{10}{3}}$$

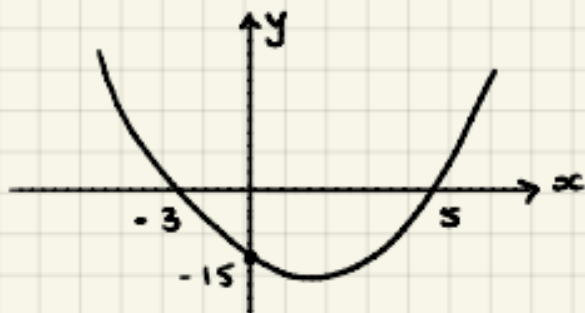
$$x = 2 \pm \sqrt{\frac{10}{3}} \text{ A1}$$

$$(ii) y = 3x^2 - 12x + 2$$

$$y = 3(x-2)^2 - 10 \text{ B1}$$



3. (a)



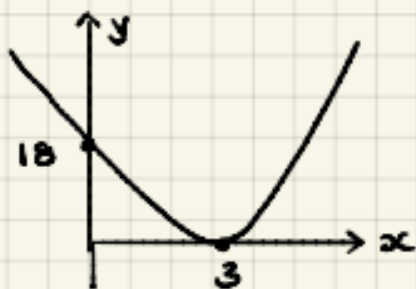
$$y = k(x+3)(x-5) \quad \text{M1}$$

$$-15 = k(3)(-5) \Rightarrow k = 1 \quad \text{A1}$$

$$y = (x+3)(x-5)$$

$$y = x^2 - 2x - 15 \quad \text{A1}$$

(b)



$$y = k(x-3)^2 \quad \text{M1}$$

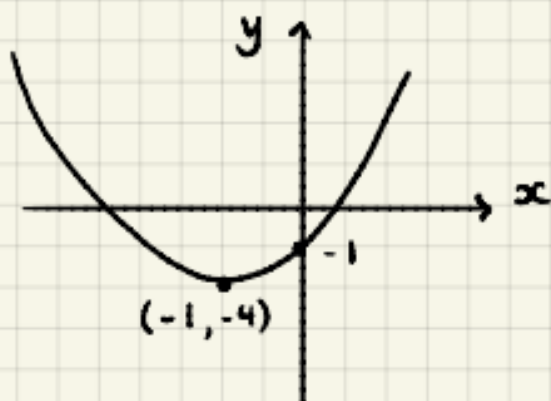
$$18 = k(-3)^2 \Rightarrow k = 2 \quad \text{A1}$$

$$y = 2(x-3)^2$$

$$y = 2(x^2 - 6x + 9)$$

$$y = 2x^2 - 12x + 18 \quad \text{A1}$$

(c)



$$y = k(x+1)^2 - 4 \quad \text{M1}$$

$$-1 = k(1)^2 - 4 \quad (0, -1)$$

$$\Rightarrow k = 3 \quad \text{A1}$$

$$y = 3(x+1)^2 - 4$$

$$y = 3(x^2 + 2x + 1) - 4$$

$$y = 3x^2 + 6x - 1 \quad \text{A1}$$

9



## 04 Simultaneous Equations

$$1. \quad 3x + 3y = -4$$

$$5x - 2y = 5$$

$$6x + 6y = -8$$

$$15x - 6y = 15 \quad \text{add}$$

M1

$$\hline 21x = 7$$

$$x = \frac{1}{3} \quad \text{A1} \quad 3\left(\frac{1}{3}\right) + 3y = -4$$

$$3y = -5$$

$$x = \frac{1}{3}, y = -\frac{5}{3} \quad \text{A1}$$

$$2. \quad y = x - 6$$

$$\frac{1}{2}x - y = 4$$

$$\frac{1}{2}x - (x - 6) = 4 \quad \text{M1}$$

$$\frac{1}{2}x - x + 6 = 4$$

$$-\frac{1}{2}x = -2$$

$$x = 4 \quad \text{A1} \quad y = 4 - 6$$

$$y = -2$$

$$x = 4, y = -2 \quad \text{A1}$$

$$3. \quad 3x^2 - x - y^2 = 0 \quad x + y = 1$$

$$3x^2 - x - (1-x)^2 = 0 \quad \text{MI} \quad y = 1-x$$

$$3x^2 - x - (1 - 2x + x^2) = 0$$

$$3x^2 - x - 1 + 2x - x^2 = 0$$

$$2x^2 + x - 1 = 0 \quad \text{AI}$$

$$(2x-1)(x+1) = 0$$

$$x = 1/2 \quad x = -1 \quad \text{AI}$$

$$y = 1 - 1/2 \quad y = 1 - (-1)$$

$$x = 1/2 \quad y = 1/2 \quad \text{AI} \quad x = -1, y = 2 \quad \text{AI}$$

□ II

### 85 Inequalities

$$1. \quad 3(1-2t) \leq t-4$$

$$3 - 6t \leq t - 4$$

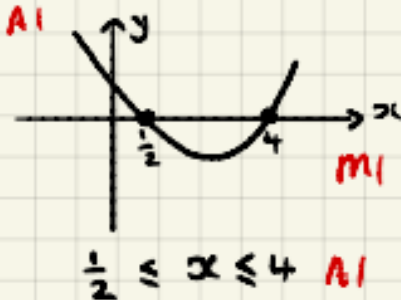
$$7 \leq 7t \quad \text{MI}$$

$$t \geq 1 \quad \text{AI}$$

$$2. \quad 2x^2 - 9x + 4 \leq 0$$

$$(2x-1)(x-4) \leq 0 \quad \text{MI}$$

$$\text{CVs } x = 1/2 \quad x = 4 \quad \text{AI}$$



$$3. \quad 2y + 3 < 3y(y-2)$$

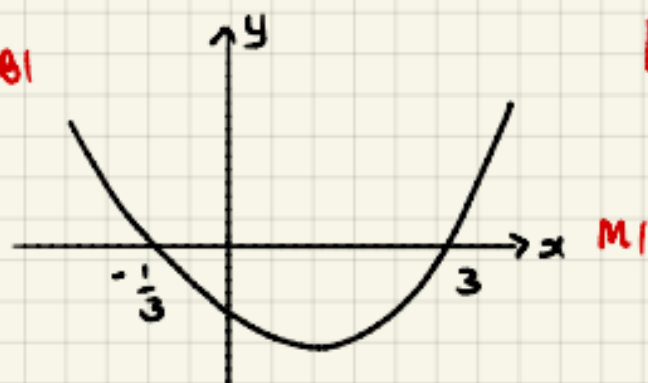
$$2y + 3 < 3y^2 - 6y$$

$$0 < 3y^2 - 8y - 3 \quad \text{MI}$$

$$3y^2 - 8y - 3 > 0$$

$$(3y+1)(y-3) > 0 \quad \text{MI}$$

$$\text{CVs } y = -1/3 \quad y = 3 \quad \text{AI}$$

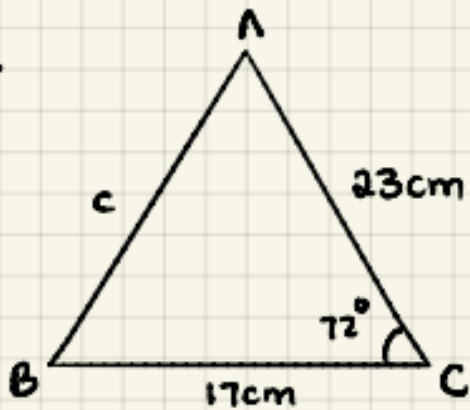


$$y < -1/3 \quad \text{or} \quad y > 3 \quad \text{AI}$$

□ II

# E1 Triangle Geometry

1.



$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = 17^2 + 23^2 - 2(17)(23) \cos 72^\circ$$

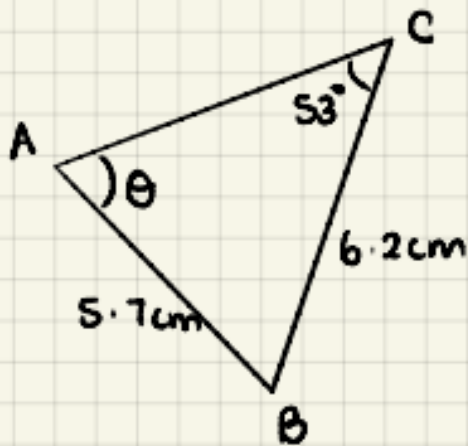
$$c^2 = 576.35$$

$$AB = 24.0 \text{ cm}$$

M1

A1

2.



$$\frac{\sin \theta}{6.2} = \frac{\sin 53}{5.7}$$

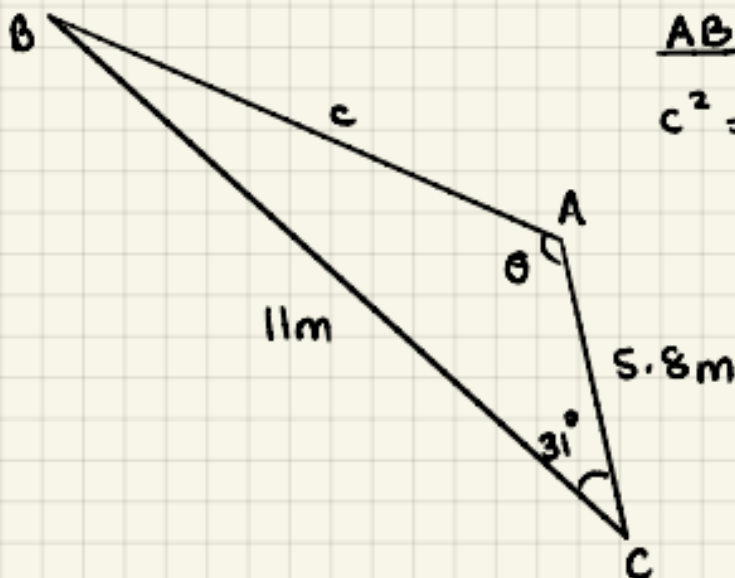
M1

$$\theta = \sin^{-1} \left( \frac{6.2 \sin 53}{5.7} \right)$$

$$\theta = 60.3^\circ$$

A1

3.



AB

$$c^2 = 5.8^2 + 11^2 - 2(5.8)(11) \cos 31$$

$$c^2 = 45.27$$

M1

$$AB = 6.7 \text{ m}$$

A1

$$\cos \theta = \frac{5.8^2 + 6.7^2 - 11^2}{2(5.8)(6.7)}$$

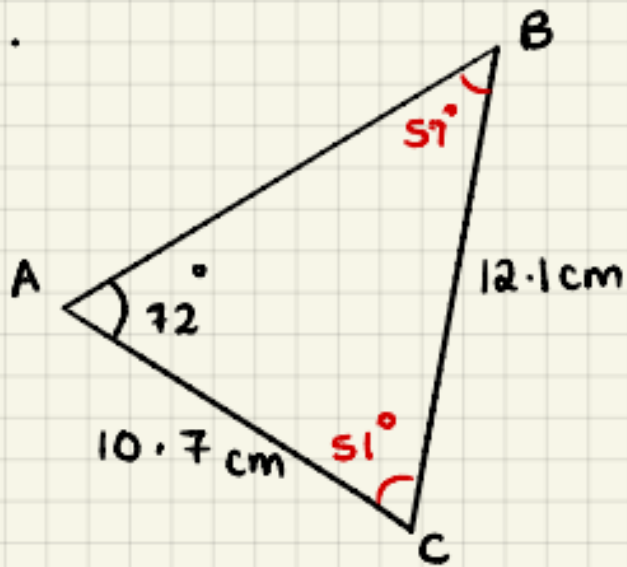
M1

$$\theta = \cos^{-1}(-0.546)$$

A1

$$\theta = 123^\circ$$

4.



$$\frac{\sin B}{10.7} = \frac{\sin 72^\circ}{12.1} \quad \text{M1}$$

$$B = \sin^{-1} \left( \frac{10.7 \sin 72^\circ}{12.1} \right)$$

$$B = 51^\circ \quad \text{A1}$$

$$A = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} (10.7)(12.1) \sin 51^\circ \quad \text{M1}$$

$$= 50.3 \text{ cm}^2 \quad \text{A1}$$

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## Year 12 Initial Test for Mathematics

Write out the solutions to each of the following questions.  
Show full working, **without** the use of a calculator.

### Practice 2 (No Calculator)

#### B1 Indices

|  |   |                                   |   |
|--|---|-----------------------------------|---|
| 1. Evaluate $\left(3\frac{3}{8}\right)^{-1/3}$ | 2. Express in the form $x^k$<br>$\frac{\sqrt{x} \times \sqrt[5]{x}}{x^2}$ | 3. Solve $3^{3x-2} = \sqrt[3]{9}$ | 4. Solve $\left(\frac{1}{2}\right)^{1-x} = \left(\frac{1}{8}\right)^{2x}$ |
|--|---|-----------------------------------|---|

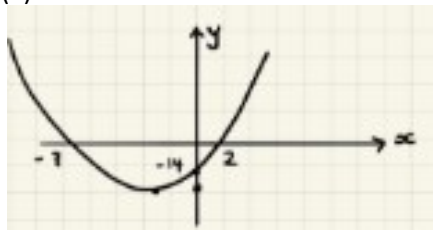
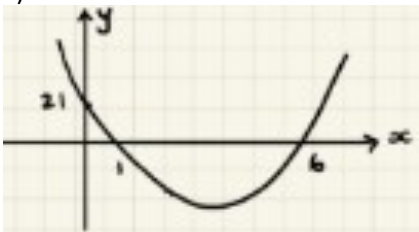
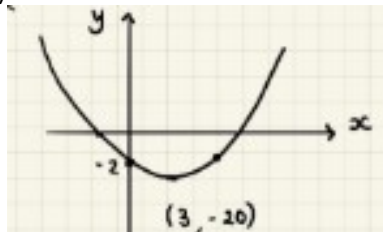
#### B2 Surds

|                         |   |   |  |
|-------------------------|---|---|--|
| 1. Simplify $\sqrt{80}$ | 2. Expand and simplify $(7 - 3\sqrt{5})(3\sqrt{5} - 2)$ | 3. Rationalise the denominator<br>$\frac{7}{5\sqrt{3}}$ | 4. Rationalise the denominator<br>$\frac{3 + 5\sqrt{11}}{7 - \sqrt{11}}$ |
|-------------------------|---|---|--|

#### B3 Quadratics

|   |                                |                                     |
|---|--------------------------------|-------------------------------------|
| 1. Solve the following quadratic equations by factorising and use your solutions to sketch the related quadratic graph, labelling all intersections with the coordinate axis. |                                |                                     |
| (a) (i) $x^2 - 13x + 40 = 0$  | (b) (i) $x^2 + 5x = 0$         | (c) (i) $6x^2 + 5x - 4 = 0$         |
| (a) (ii) Sketch $y = x^2 - 13x + 40$  | (b) (ii) Sketch $y = x^2 + 5x$ | (c) (ii) Sketch $y = 6x^2 + 5x - 4$ |

|   |  |  |
|---|--|--|
| 2. Solve the following quadratic equations by completing the square and use your solutions to sketch the related quadratic graph, labelling all intersections with the coordinate axis and turning point. |  |  |
| (a) (i) $x^2 + 2x - 20 = 0$   | (b) (i) $-11 + 8x - x^2 = 0$                                     | (c) (i) $3x^2 - 18x + 2 = 0$                                     |
| (ii) Write $y = x^2 + 2x - 20$ in the form $y = a(x + b)^2 + c$   | (ii) Write $y = -11 + 8x - x^2$ in the form $y = a(x + b)^2 + c$ | (ii) Write $y = 3x^2 - 18x + 2$ in the form $y = a(x + b)^2 + c$ |
| (iii) Sketch $y = x^2 + 2x - 20$  | (iii) Sketch $y = -11 + 8x - x^2$                                | (iii) Sketch $y = 3x^2 - 18x + 2$                                |

|  |  |   |
|--|--|---|
| 3. Evaluate the equation of the following quadratics, giving your answer in the form $y = ax^2 + bx + c$ |  |   |
| (a)                   | (b)  | (c)  |

**B4 Simultaneous Equations**

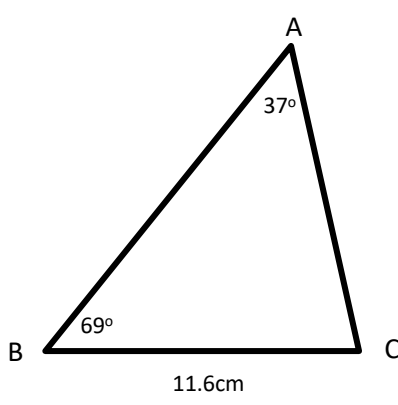
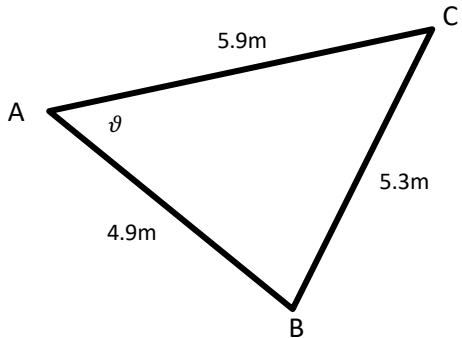
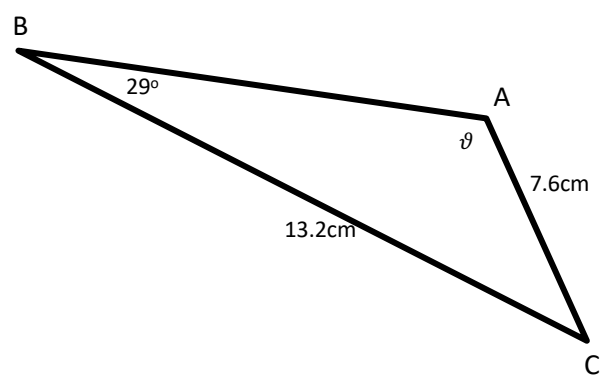
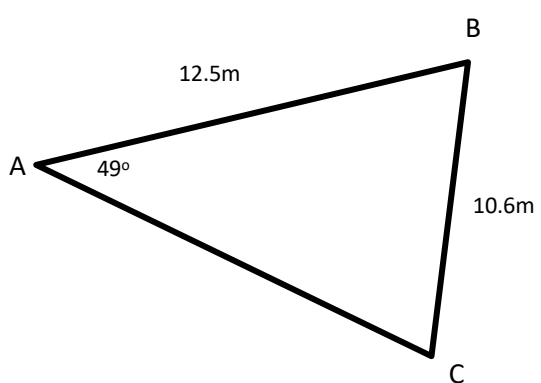
|  |  |   |
|--|--|---|
| <b>1.</b> Solve<br>$3x - 4y = 16$ $2x + 12y = 7$ | <b>2.</b> Solve<br>$3y = 2x - 8$ $4x + y = -5$ | <b>3.</b> Solve<br>$3x^2 - xy + y^2 = 36$ $x - 2y = 10$ |
|--|--|---|

**B5 Inequalities**

Find the set of values for which...

|                                      |                               |                                    |
|--------------------------------------|-------------------------------|------------------------------------|
| <b>1.</b> $4(5 - 2y) \geq 3(7 - 2y)$ | <b>2.</b> $2x^2 - 5x - 3 > 0$ | <b>3.</b> $x(2x + 1) \leq x^2 + 6$ |
|--------------------------------------|-------------------------------|------------------------------------|

**E1 Triangle Geometry (Calculator)**

|   |  |
|---|--|
| <b>1.</b> Calculate the length AB<br>                                   | <b>2.</b> Calculate the angle $\vartheta$<br>         |
| <b>3.</b> Calculate the length AB and the obtuse angle $\vartheta$<br> | <b>4.</b> Calculate the area of the triangle ABC<br> |

## Practice Test 2

### B1 Indices

$$1. \left(3\frac{2}{8}\right)^{-1/3} = \left(\frac{27}{8}\right)^{-1/3} \quad \text{M1}$$

$$= \left(\frac{8}{27}\right)^{1/3}$$

$$= \frac{2}{3} \quad \text{A1}$$

$$2. \frac{\sqrt{x} \times \sqrt[3]{x}}{x^2}$$

$$= \frac{x^{1/2} \times x^{1/3}}{x^2} \quad \text{M1}$$

$$= \frac{x^{5/6}}{x^2} = x^{-7/6} = x^{-1\frac{1}{6}} \quad \text{A1}$$

$$3. 3^{3x-2} = \sqrt[3]{9}$$

$$3^{3x-2} = (3^2)^{1/3} \quad \text{M1}$$

$$3^{3x-2} = 3^{2/3}$$

$$3x-2 = \frac{2}{3} \quad \text{M1}$$

$$3x = \frac{8}{3} \Rightarrow x = \frac{8}{9} \quad \text{A1}$$

$$4. \left(\frac{1}{2}\right)^{1-2x} = \left(\frac{1}{8}\right)^{2x}$$

$$(2^{-1})^{1-2x} = (2^{-3})^{2x} \quad \text{M1}$$

$$2^{-1+2x} = 2^{-6x}$$

$$-1+2x = -6x \quad \text{M1}$$

$$7x = 1$$

$$x = \frac{1}{7} \quad \text{A1}$$

□□

### B2 Surds

$$1. \sqrt{80}$$

$$= \sqrt{16 \times 5}$$

$$= 4\sqrt{5} \quad \text{A1}$$

$$2. (7-3\sqrt{5})(3\sqrt{5}-2)$$

$$= 21\sqrt{5} - 14 - 45 + 6\sqrt{5} \quad \text{M1 A1}$$

$$= 27\sqrt{5} - 59 \quad \text{A1}$$

$$3. \frac{7}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \quad \text{M1}$$

$$= \frac{7\sqrt{3}}{15} \quad \text{A1}$$

$$4. \frac{3+5\sqrt{11}}{7-\sqrt{11}} \times \frac{(7+\sqrt{11})}{(7+\sqrt{11})} \quad \text{M1}$$

$$= \frac{21+3\sqrt{11}+35\sqrt{11}+55}{49-11} \quad \text{A1}$$

$$= \frac{76+38\sqrt{11}}{38} \quad \text{A1}$$

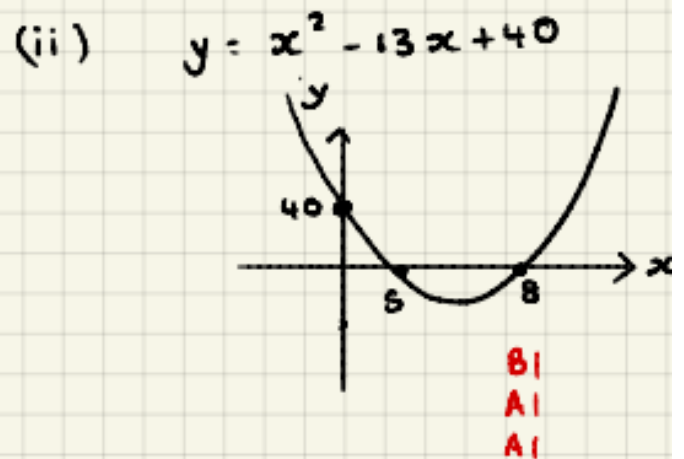
$$= 2+\sqrt{11} \quad \text{A1}$$

□□

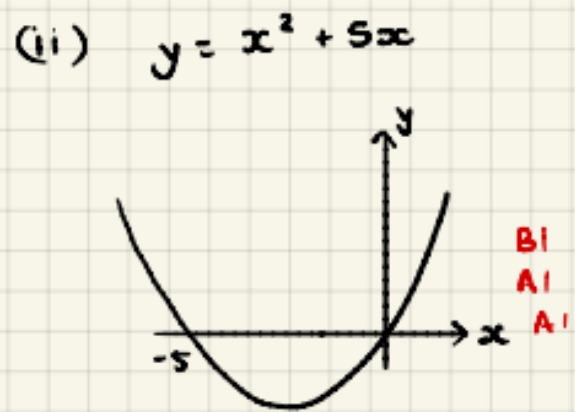


### B3 Quadratics

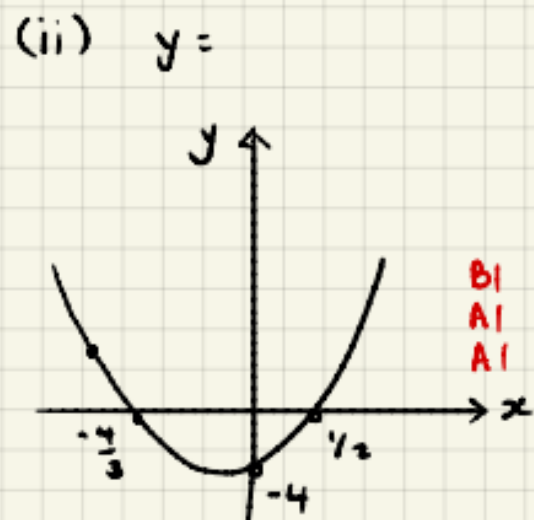
1. (a) (i)  $x^2 - 13x + 40 = 0$   
 $(x-8)(x-5) = 0$  M1  
 $x = 8 \quad x = 5$  A1



(b) (i)  $x^2 + 5x = 0$   
 $x(x+5) = 0$  M1  
 $x = 0 \quad x = -5$  A1



(c) (i)  $6x^2 + 5x - 4 = 0$   
 $(3x+4)(2x-1) = 0$  M1  
 $x = -4/3 \quad x = 1/2$  A1



15

B1 shape, location related to axes

A1 intersections x-axis

A1 intersections y-axis

$$2. (a) (i) x^2 + 2x - 20 = 0$$

$$(x+1)^2 - 1 - 20 = 0 \quad \text{M1}$$

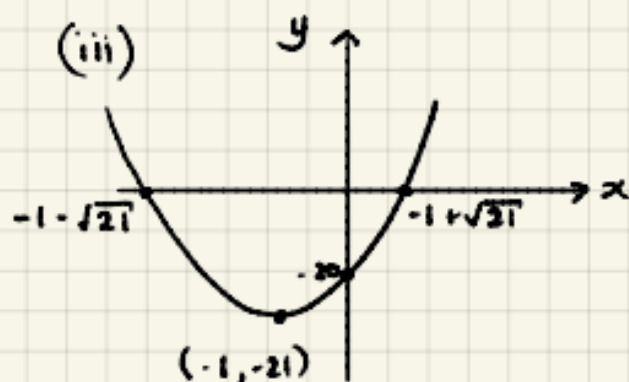
$$(x+1)^2 = 21$$

$$x+1 = \pm\sqrt{21}$$

$$x = -1 \pm \sqrt{21} \quad \text{A1}$$

$$(ii) y = x^2 + 2x - 20$$

$$y = (x+1)^2 - 21 \quad \text{B1}$$



Graphs

B1 Shape

A1 Vertex

A1 intersections x-axis

A1 intersections y-axis

$$(b) (i) -11 + 8x - x^2 = 0$$

$$-(x^2 - 8x + 11) = 0 \quad \text{M1}$$

$$-[(x-4)^2 - 16 + 11] = 0 \quad \text{M1}$$

$$5 - (x-4)^2 = 0$$

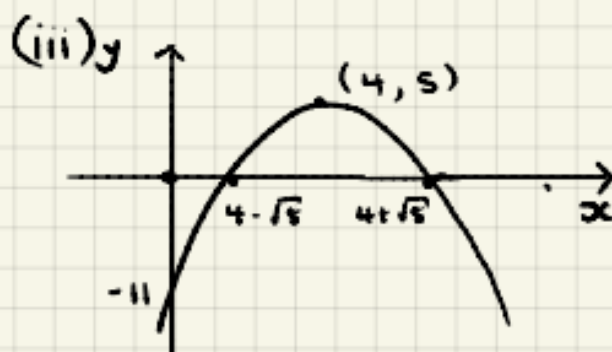
$$(x-4)^2 = 5$$

$$x-4 = \pm\sqrt{5}$$

$$x = 4 \pm \sqrt{5} \quad \text{A1}$$

$$(ii) y = -11 + 8x - x^2$$

$$y = 5 - (x-4)^2 \quad \text{B1}$$



$$(c) (i) 3x^2 - 18x + 2 = 0$$

$$3[x^2 - 6x + \frac{2}{3}] = 0 \quad \text{M1}$$

$$3[(x-3)^2 - 9 + \frac{2}{3}] = 0 \quad \text{M1}$$

$$3[(x-3)^2 - \frac{25}{3}] = 0$$

$$3(x-3)^2 - 25 = 0$$

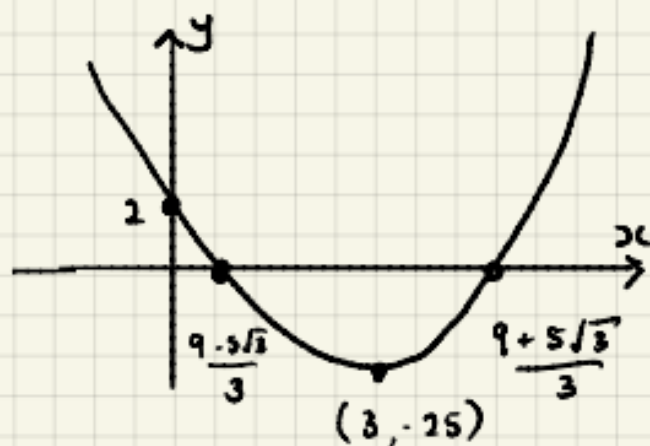
$$3(x-3)^2 = 25$$

$$x-3 = \pm\frac{5}{\sqrt{3}}$$

$$x = \frac{9 \pm 5\sqrt{3}}{3} \quad \text{A1}$$

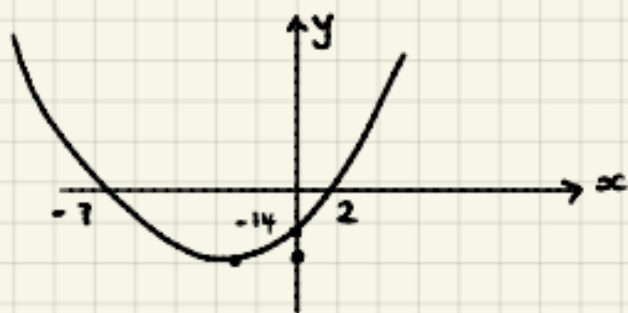
$$(ii) y = 3x^2 - 18x + 2$$

$$y = 3(x-3)^2 - 25 \quad \text{B1}$$



**23**

3. (a)



$$y = k(x+7)(x-2) \quad \text{M1}$$

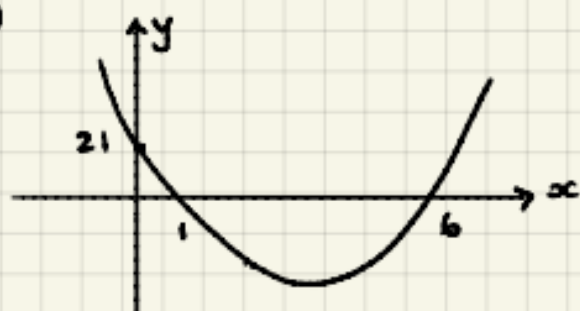
$$-14 = k(7)(-2)$$

$$k = 1 \quad \text{A1}$$

$$y = (x+7)(x-2)$$

$$y = x^2 + 5x - 14 \quad \text{A1}$$

(b)



$$y = k(x-1)(x-6) \quad \text{M1}$$

$$21 = k(-1)(-6)$$

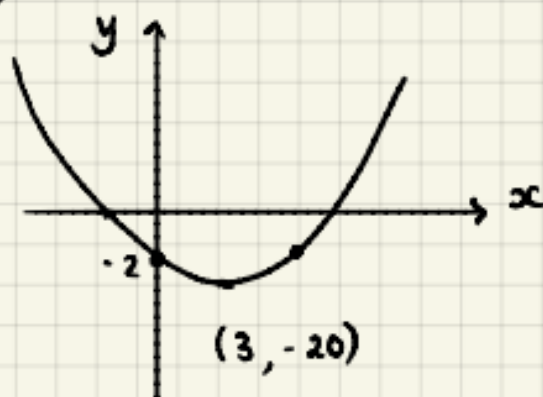
$$\Rightarrow k = \frac{21}{6} = \frac{7}{2} \quad \text{A1}$$

$$y = \frac{7}{2}(x-1)(x-6)$$

$$y = \frac{7}{2}(x^2 - 7x + 6)$$

$$y = \frac{7x^2}{2} - \frac{49x}{2} + 21 \quad \text{A1}$$

(c)



$$y = k(x-3)^2 - 20 \quad \text{M1}$$

$$-2 = k(-3)^2 - 20$$

$$18 = k(9)$$

$$k = 2 \quad \text{A1}$$

$$y = 2(x-3)^2 - 20$$

$$y = 2(x^2 - 6x + 9) - 20$$

$$y = 2x^2 - 12x - 2 \quad \text{A1}$$

9

## B4. Simultaneous Equations

1.  $3x - 4y = 16$

$$2x + 12y = 7$$

$$9x - 12y = 48$$

$$\underline{2x + 12y = 7}$$

$$11x = 55$$

$$x = 5 \quad \text{AI}$$

M1

$$3x - 4y = 16$$

$$15 - 4y = 16$$

$$-1 = 4y$$

$$y = -1/4$$

$$x = 5, y = -1/4 \quad \text{AI}$$

2.  $3y = 2x - 8 \Rightarrow 2x = 3y + 8$

$$4x + y = -5$$

$$4x = 6y + 16$$

M1

$$6y + 16 + y = -5$$

$$7y = -21$$

$$y = -3$$

$$2x = 3y + 8$$

$$2x = 3(-3) + 8$$

$$x = -1/2 \quad \text{AI}$$

$$x = -1/2, y = -3 \quad \text{AI}$$

3.  $3x^2 - xy + y^2 = 36$

$$x - 2y = 10 \Rightarrow x = 2y + 10$$

$$3(2y + 10)^2 - (2y + 10)y + y^2 = 36 \quad \text{M1}$$

$$3(4y^2 + 40y + 100) - y(2y + 10) + y^2 = 36$$

$$12y^2 + 120y + 300 - 2y^2 - 10y + y^2 = 36$$

$$11y^2 + 110y + 264 = 0$$

$$y^2 + 10y + 24 = 0 \quad \text{AI}$$

$$(y + 6)(y + 4) = 0$$

$$y = -6 \quad y = -4 \quad \text{M1}$$

$$x = 2(-6) + 10 \quad x = 2(-4) + 10$$

$$x = -2 \quad x = 2$$

$$x = -2, y = -6 \quad \text{AI} \quad x = 2, y = -4 \quad \text{AI}$$



## BS Inequalities

1.  $4(5-2y) > 3(7-2y)$

$$20 - 8y > 21 - 6y \quad \text{MI}$$

$$-1 > 2y$$

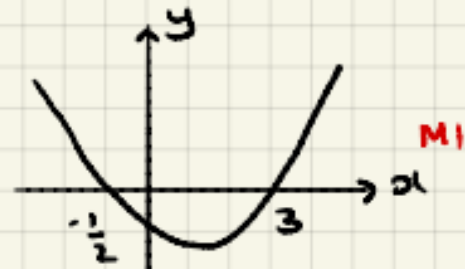
$$-1/2 > y$$

$$y < -1/2 \quad \text{AI}$$

2.  $2x^2 - 5x - 3 > 0$

$$(2x+1)(x-3) > 0 \quad \text{MI}$$

CVs  $x = -1/2$   $x = 3$  AI



$$x < -1/2 \text{ or } x > 3 \quad \text{AI}$$

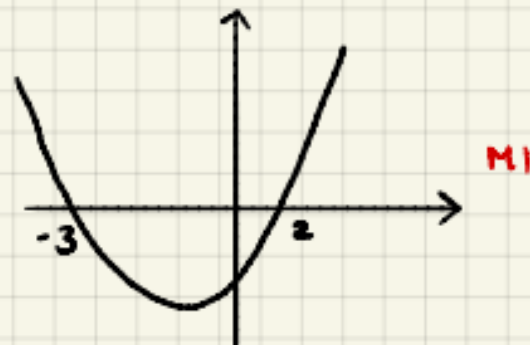
3.  $x(2x+1) \leq x^2 + 6$

$$2x^2 + x \leq x^2 + 6 \quad \text{MI}$$

$$x^2 + x - 6 \leq 0$$

$$(x+3)(x-2) \leq 0 \quad \text{MI}$$

CVs  $x = -3$   $x = 2$  AI

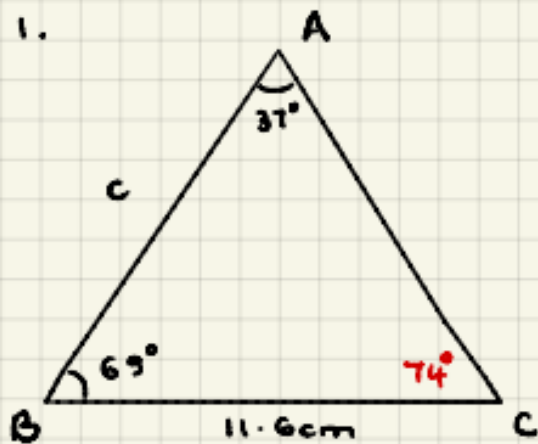


$$-3 \leq x \leq 2 \quad \text{AI}$$



# E1 Triangle Geometry

1.

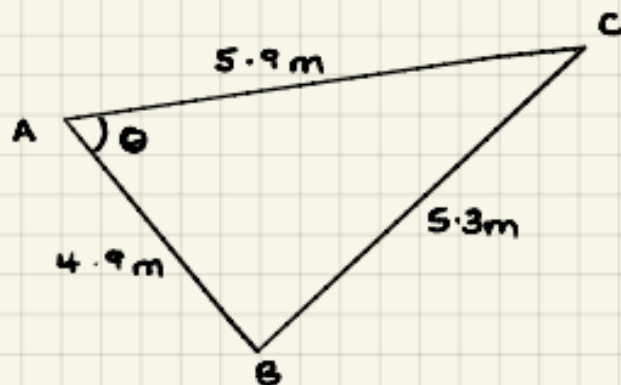


$$\frac{c}{\sin 74^\circ} = \frac{11.6}{\sin 37^\circ}$$

$$c = \frac{11.6 \sin 74^\circ}{\sin 37^\circ} \quad \text{M1}$$

$$c = 18.5 \text{ cm} \quad \text{A1}$$

2.



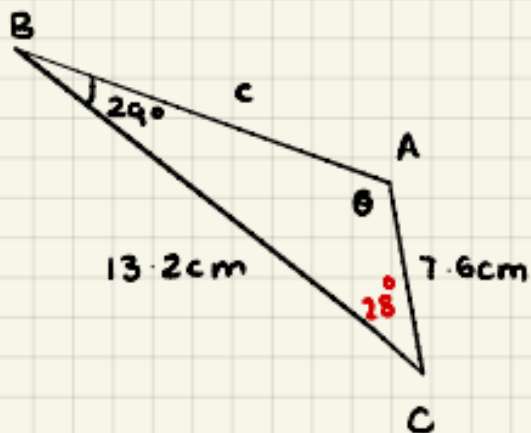
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos \theta = \frac{5.9^2 + 4.9^2 - 5.3^2}{2(5.9)(4.9)} \quad \text{M1}$$

$$\cos \theta = 0.53148$$

$$\theta = 57.9^\circ \quad \text{A1}$$

3.



$\approx$

$$\frac{\sin \theta}{13.2} = \frac{\sin 29^\circ}{7.6}$$

$$\sin \theta = \frac{13.2 \sin 29^\circ}{7.6} \quad \text{M1}$$

$$\sin \theta = 0.8420$$

$$\theta = 57.4^\circ$$

$$\text{obtuse} \Rightarrow \theta = 123^\circ \quad \text{A1}$$

AE

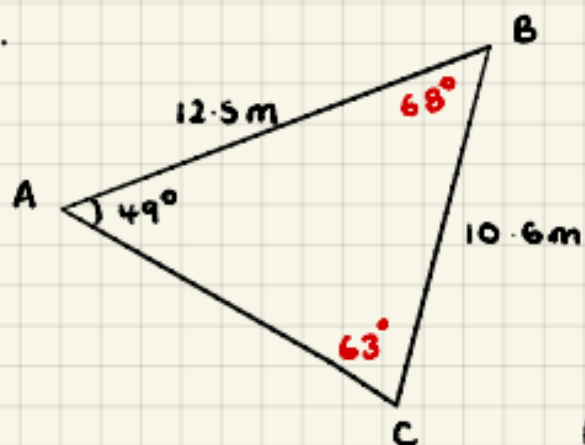
$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = 13.2^2 + 7.6^2 - 2(13.2)(7.6) \cos 28 \quad \text{M1}$$

$$c^2 = 54.8 \Rightarrow c = 7.4 \text{ cm} \quad \text{A1}$$



4.



$$\frac{\sin C}{12.5} = \frac{\sin 49^\circ}{10.6}$$

$$\sin C = \frac{12.5 \sin 49^\circ}{10.6} \quad M1$$

$$C = 63^\circ \quad A1$$

$$\text{Area} = \frac{1}{2} ab \sin C$$

$$\text{Area} = \frac{1}{2} (12.5)(10.6) \sin 68^\circ \quad M1$$

$$\text{Area} = 61.5 \text{ m}^2 \quad A1$$

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